Safety in Numbers: Math in Your Head

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I've done lots of calculations in the outdoors, and I've never used a calculator to do a single one. I make a point of performing only simple calculations that fit easily into my old brain. This essay provides some examples of math that is both easy and useful.

We'll start with simple math on simple problems. Later on, the essay uses some math techniques that move beyond elementary school and well into middle school. If you feel uncomfortable with any of the math, please put down this essay immediately and move on to one that you find more pleasurable.

Walking Times

In "How Fast and How Far", I explained that I estimate my walking times by assuming that I walk at about three miles per hour, and then adjust for elevation gain. On a great trail when I'm fresh, my pace might in fact be 3.3 mph. And on a nasty trail when I'm pooped out, it might be 2.7 mph. But when I calculate, I'll always use 3 mph, because it makes the math so easy.

Three miles per hour is one mile every twenty minutes. That is a half mile in ten minutes, and a quarter mile in five minutes. So if I see a sign that says that I'm 1¾ miles from my car, I'll break that into pieces: (1 mile = 20 min) + (½ mile = 10 min) + (¼ mile = 5 min), so I'm 35 minutes from the car.

If I see that sign at 4:19, then I'll estimate that I'll be at my car around 4:54. If I later see a sign that says that I am one mile to the parking area, I'll guess that I'm about 20 minutes out, and that the time is ballpark 4:34. If I'm strong and fast, then the time might really be 4:32, and if I'm tired and slow, it might be 4:36. But it is usually pretty close to my guess.

But even if I knew that my pace were really 2.7 mph, I would still calculate with 3 mph because it makes the calculations easy. When it really matters, I'll adjust my times by adding an extra ten percent.

If the trail sign gives the distances in decimal, then I recall that I walk a mile in 20 minutes, and a tenth of a mile in two minutes. So if the sign says that I'm 2.3 miles away, then I expect to be there in about 46 minutes. And if the sign says that I'm 17.7 miles away, then I expect to be there in about 6 hours – it would be pretty silly to worry about 2 minutes here and there over that long time and big distance!

I usually estimate that I gain elevation at the rate of about 1000 vertical feet per hour. That number is really convenient for the big picture: I know that gaining 4700 feet will take me about 4 2/3 hours. Up close, that rate is the same as 500 feet every 30 minutes, 250 feet every 15 minutes, or 50 feet every 3 minutes. With numbers this simple, I never get close to doing fancy calculations (9 times 7 is 63, carry the 7, …). I break a problem into little pieces, find the answers, and then put them back together again.

Metric Maps
Every now and then I end up walking on a map that measures distances in kilometers and heights in meters. I could use an English (or Imperial) to Metric converter, and find that my 3 mph is 4.83 kph, and 1000 vertical feet per hour is 304.8 meters per hour. I probably could memorize those numbers, but working with them would be a pain. Amazingly, I find that when I am walking on a metric map, my base pace speeds up just a tad to 5 kph. This rate is one km every 12 minutes, half a km in 6 minutes, and a quarter km every 3 minutes.

Of course, to make up for this increase in my pace, my ascension slows by 1½ percent down to 300 meters per hour, or 100 meters every 20 minutes. The math is now as easy as the walking. I know some people who ascend 305 meters per hour, but I can't keep up with those wild people.

Metric Snow on my Tarp
In "A Catskill Christmas" I told the story about spending Christmas Night of 2002 on top of Slide Mountain in some interesting weather. Early in the evening, I calculated how much snow might end up on my tarp, and estimated that it might be as much as half a ton. That motivated me to get up often to clear the snow.

When I did the calculation that night, I was pretty sure that it was accurate enough. My brain lives in the English system of yards, miles, tons and pints, and I did the calculation in inches, feet and pounds. But just to make sure that my answer was correct, later that night I did the calculation all over in the metric system, and came up with about the same answer.

I'll tell that story again, using the metric system from the very start. The forecast that evening was for twelve below (centigrade), with 50 kilometer/hour winds dumping a meter of snow by later in the evening. I was under a 3-meter by 3-meter tarp, and I figured that it might accumulate 60 centimeters of snow by the end of the evening. How much would that snow on top of me weigh?

I know that a cubic centimeter of water weighs one gram, and that a 10 cm cube of water weighs one kilogram. A square meter of water 10cm deep therefore weighs 100 kg. But light, fluffy snow is about one-tenth as dense as water, so 60 cm of that white stuff across a square meter would weigh the same as 6 cm of water, which is 60 kg. My 3-meter-square tarp has a total of 9 square meters. Multiplying 9 m^2 times 60 kg/m^2 gives a total of 540 kg, or just over half a metric ton.

When you get the same answer in two very different ways, you have more confidence that it is correct. This was ample motivation for me to emerge from a warm sleeping bag many times to ensure that I wasn't crushed under snow.

Increasing Accuracy

In the English system, I found that the weight of the snow was “about half a ton”; in the metric system, it is still “about half a ton”. For that evening, those answers were “close enough”. But my previous answer was about 960 pounds, while this one is 540 kg, or 1190.5 pounds. What gives?

The crux of the problem is the size of the tarp. In the English system, I said that it was 9-feet square, while in the metric system I said it was 3-meters square. Close, but not close enough. Three meters is not 9 feet, but rather 9.84 feet. Oops.

We can use that fact to fix our previous estimate. I know that a meter is about 10 percent longer than a yard (more precisely, 9.361 percent, but 10 percent is good enough). Thus each side of the real 9-foot-square tarp is only about 90 percent (0.9 times) as long as the fictitious 3-meter-square tarp. I therefore square 0.9 to get 0.81, and round that to 0.8, or eighty percent. I’ll then subtract about 20 percent from the previous answer as a correction: 20 percent of 540 is about 110, and subtracting that from the original gives 430. We find that 430 kilograms is about 948 pounds, which is pretty close to the original 960 pounds.

Rather than fixing our flawed estimate, we could just perform the whole calculation with more accuracy, using mathematical tricks. We know that the tarp is a square, 2.7 m on a side, and that a 10 cm cube of water weighs 1 kg. We assume that light snow is about one-tenth as dense as water, and thus a square meter of snow 60cm deep weighs about 60kg. We therefore want to solve this mathematical equation:

$$(2.7\ \mathrm{m})^2 \times 60\ \mathrm{kg/m^2}$$

No way can I do that in my head! But I can break it into simpler pieces. The first term is

$$(3 \times 0.9)^2 = 3^2 \times 0.9^2 = 9 \times .81 \sim 9 \times .8 = 7.2$$

Alternatively, being a math geek, I’m familiar with powers of 3: 1, 3, 9, 27, 81, 243, 729, …. I know that 27 squared is 729, so I could (in the right mood) see at a glance that 2.7 squared is 7.29. But in either case, the tarp is about 7.2 m^2. We can re-express to multiply that by 60 $\mathrm{kg/m^2}$ as follows:

$$7.2 \times 60 = 720 \times 3/5 = 3 \times (720 / 5) = 3 \times 144 = 432$$

To divide 720 by 5, I can just toss the zero and multiply by two instead (that is, dividing by five is the same as dividing by ten and then multiplying by two). With more than a little practice, some people enjoy doing such calculations in their heads.

Techniques

Most people should find these rules easy to employ.

Choose convenient numbers for your rules of thumb. What you lose in accuracy is made up for by ease of use.

If accuracy is all that important, then add in an error term (for example, if you walk at 2.7 mph, calculate for 3 mph, then add ten percent more time to your final answer).

Calculate twice in two different ways to check important numbers.

If you are particularly fond of math, try some of these techniques.

Re-express quantities to convenient units.

Round values to one or two digits, as in slide rule arithmetic.

Tastefully choose when and how to round.
Re-order calculations when it helps.
Exploit algebraic identities.
Memorize square numbers and small powers of 2, 3 and 5.